How long is Galileo's *punctus*?

Roberto Vergara Caffarelli

1. *Piccioli, denari* and *punti*.

Some pages in the volume 72 of the Manoscritti Galileiani[1], now in the National Library in Florence, were not included (or only partly included) in the national edition of the *Opere di Galileo Galilei*, both because they are difficult to interpret and because these writings are extremely fragmentary. Stillman Drake deserves praise for first drawing attention to these unpublished notes[2], and for recognising their significance as evidence of Galileo's experiments and as a key to dating of such experiments by studying the watermarks and the handwriting.

Some of these pages of Galileo's notes on the motion of bodies include measurements of distances in which the measurement unit for length is an unspecified quantity, the *punctus*.

Drake[3] gave a value to the *punctus* based on the details taken from sheet 166r[4], which in his opinion dates back to 1606. The document shows

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[1] I have pleasure in thanking the Biblioteca Nazionale Centrale di Firenze for allowing the reproduction and publication of some pages of MS Gal 72: folios 41r, 43r, 107r, 107v,116v, 141r, 166r, 189v.

[2] Stillman Drake, *Galileo's Notes on Motion, arranged in probable order of composition and presented in reduced facsimile* (Firenze: Annali dell'Istituto e Museo di Storia della Scienza, suppl. Fasc. 2, Monografia 3, 1979).

[3]Stillman Drake, "Galileo's Experimental Confirmation of Horizontal Inertia: Unpublished Manuscripts," *Isis* , 1973, 64: 291-305; "Galileo's Work on Free Fall in 1604," *Physis*, 1974, 16:309-322, on p. 313; "Galileo's physical measurements," *Am. J. Phys*, 1986, 54:302-306. See also Stillman Drake and James MacLachlan: *Galileo's Discovery of the Parabolic Trajectory*," *Scientific American*, 1975, 232: 102-110, on p. 104.

the drawing of a quarter of a circumference of 100,000 units in radius divided into minor arcs and chords. It compares the times of the descent along successive chords using the "Law of the Fall of the Bodies" that Galileo already knew: the spaces vary as the square of the times and the sine of the angle of inclination. These measurements are also given in *puncta*: for instance, the radius *ad* (in the figure below, on the right side of the square) is given as «*ad* long[a]: pu[n]cta.180». Drake measured the distances of the segments whose length was given in *puncta* (Galileo's drawing is very inaccurate) and concluded that this unit equals just a bit less than one millimetre, more precisely 29/30 of a millimetre. He also measured the distance of the points marked on the so-called *arithmetic line* of the geometrical and military compass, presumably of Galilean origin[5], now in the *Istituto e Museo di Storia della Scienza* in Florence, finding that 180 divisions corresponded to 169 millimetres. Hence, Drake argued that Galileo must have used such measurement unit for his researches. In a later publication, he went so far as to assume that Galileo must have had a short ruler (58 mm), divided into 60 puncti that he used in his drawings and in the measurement of distances along the inclined plane.

> Galileo's calculations to the right [of the sheet 107v, see below] show how he actually got the numbers entered in the third column. In each calculation he multiplied 60 by some integer and then added a number less than 60. That is just what someone would do if he were measuring distances with a short ruler divided into 60 equal parts. Galileo certainly had such a ruler, because several diagrams in his note on motion were drawn so that a principal line contained 60 times the number of units in

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[4] Galileo Galilei, *Opere*, Edizione Nazionale, ed. Antonio Favaro, 20 vols., Vol. VIII: *Frammenti attenenti ai Discorsi intorno a due nuove scienze*, (Firenze: Barbèra, 1968), pp. 419-421.

[5] Perhaps it is the same one presented by Galileo to the Grand Duke along with the specimen of the *Operazioni del Compasso Geometrico et Militare*, printed in only 60 copies and dedicated to Cosimo de' Medici. The measurements were made by Thomas Settle in summer 1972.

which all the other lines were measured. In one note he gave “180 points” as the length of a line I measured as 174 millimeters, so that we may take Galileo’s unit as being 29/30 of a millimeter. That is almost the shortest graduation (which he called a “point”) measured on his own proportional compass at the Museum of the History of Science in Florence[6].

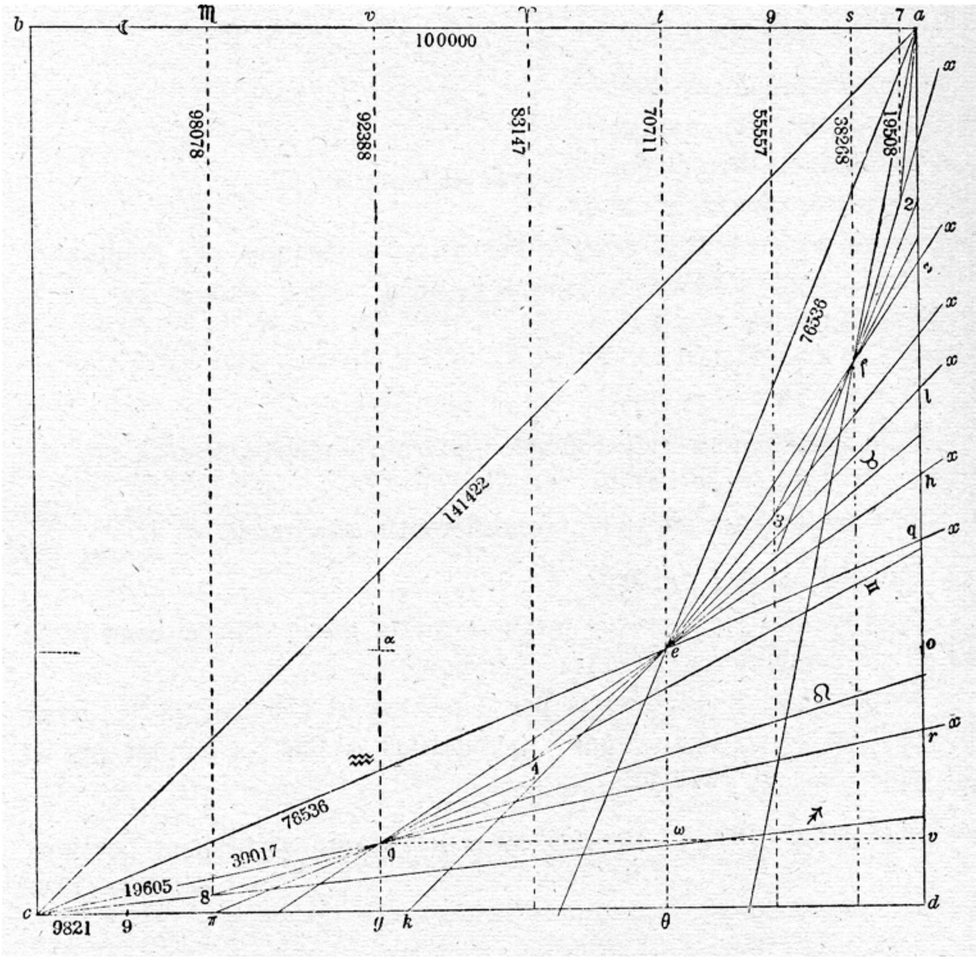
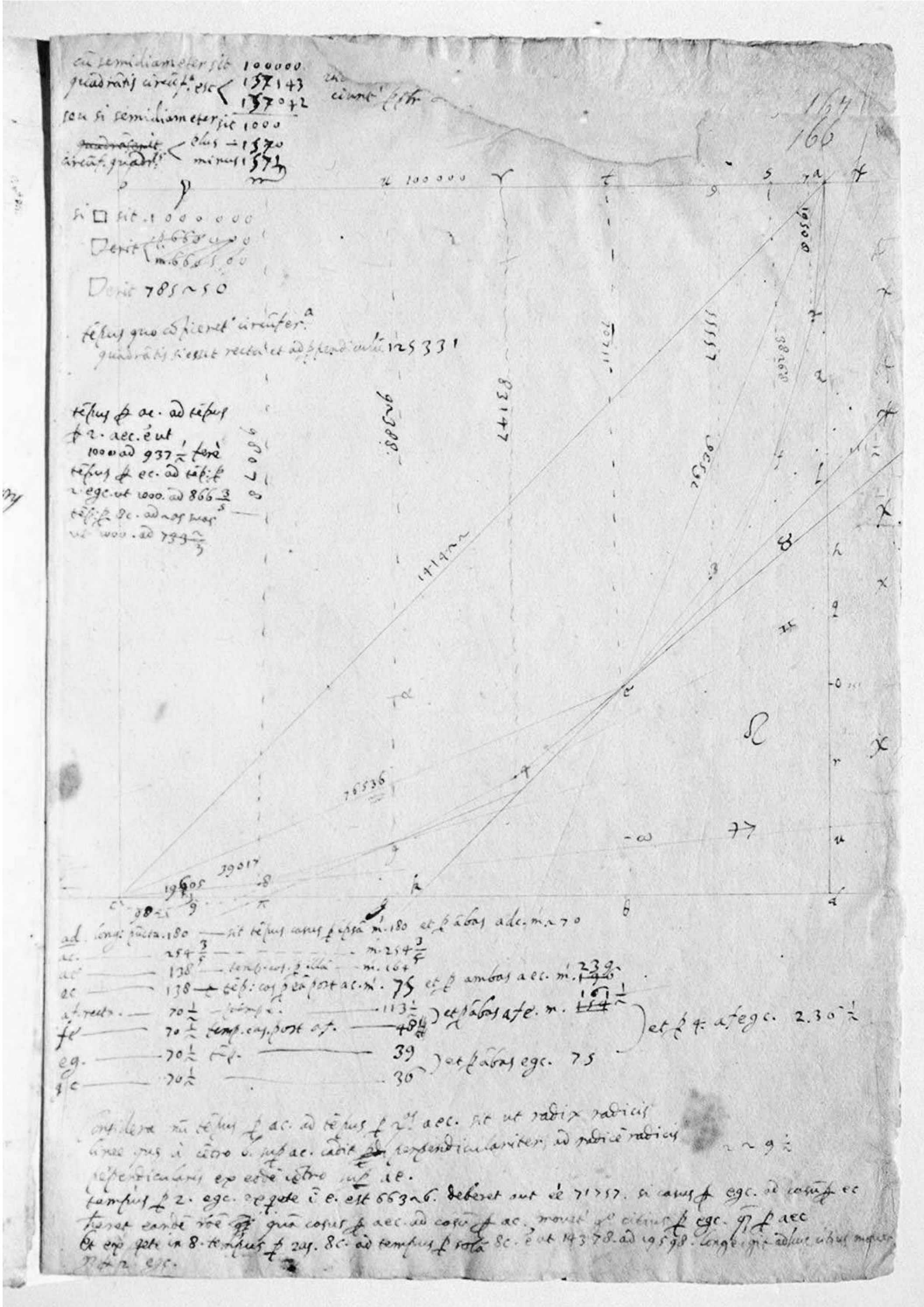


fig.1. sheet 166r as reconstructed in the National Edition

So far, all the authors who have studied these papers and have put forward their interpretations of Galileo’s notes on motion have accepted this hypothesis.

[6] Stillman Drake, “The Role of Music in Galileo's Experiments,” *Scientific American*, 1975, 235: 98-104, on p. 101. See also, *Galileo Galilei pioniere della scienza*, (Padova: Franco Muzio, 1992), p. 7.



Fig, 2. sheet 166r

Roland H. Naylor[7], for instance, says that in 1974 he made some measurements on Galileo's compass in Florence and found a mean value for the *punto* of a little bit less than 0.95 mm.

David K.Hill[8] noted that the units of the compass range between 0.939 and 0.95 millimetres: he took the fraction 236/250 of a millimetre, i.e. the equivalence of one *punto* to 0.944 mm, (approximately to 1/27 of an inch).

More recently, A. J. Hahn in a paper on Galileo's experiments, states:

> There is agreement among the historians already mentioned (based on evidence from folio 166r) that one *punto* is equal to approximately 0,94 millimeters.

It has thus been assumed for over 20 years that one *punto* is approximately equal to 0.94 millimetres.

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[7] Roland H. Naylor, "Galileo: the Search for the Parabolic Trajectory," *Annals of Science*, 1976, 33:153-172; "Galileo's Theory of Motion: processes of Conceptual Change in the Period 1604-1610," *Annals of Science*, 1977, 34:365-392. He had already regularly used the value of the *punto* as equal to 0.938 mm in the previous publication, "Galileo and the Problem of Free Fall," *British Journal for the History of Science*, 1974, 7:105-134.

[8] David K. Hill, "Dissecting Trajectories. Galileo's Early Experiments on Projectile Motion and the Law of Fall," *Isis*, 1988, 79: 646-648, on p. 648.

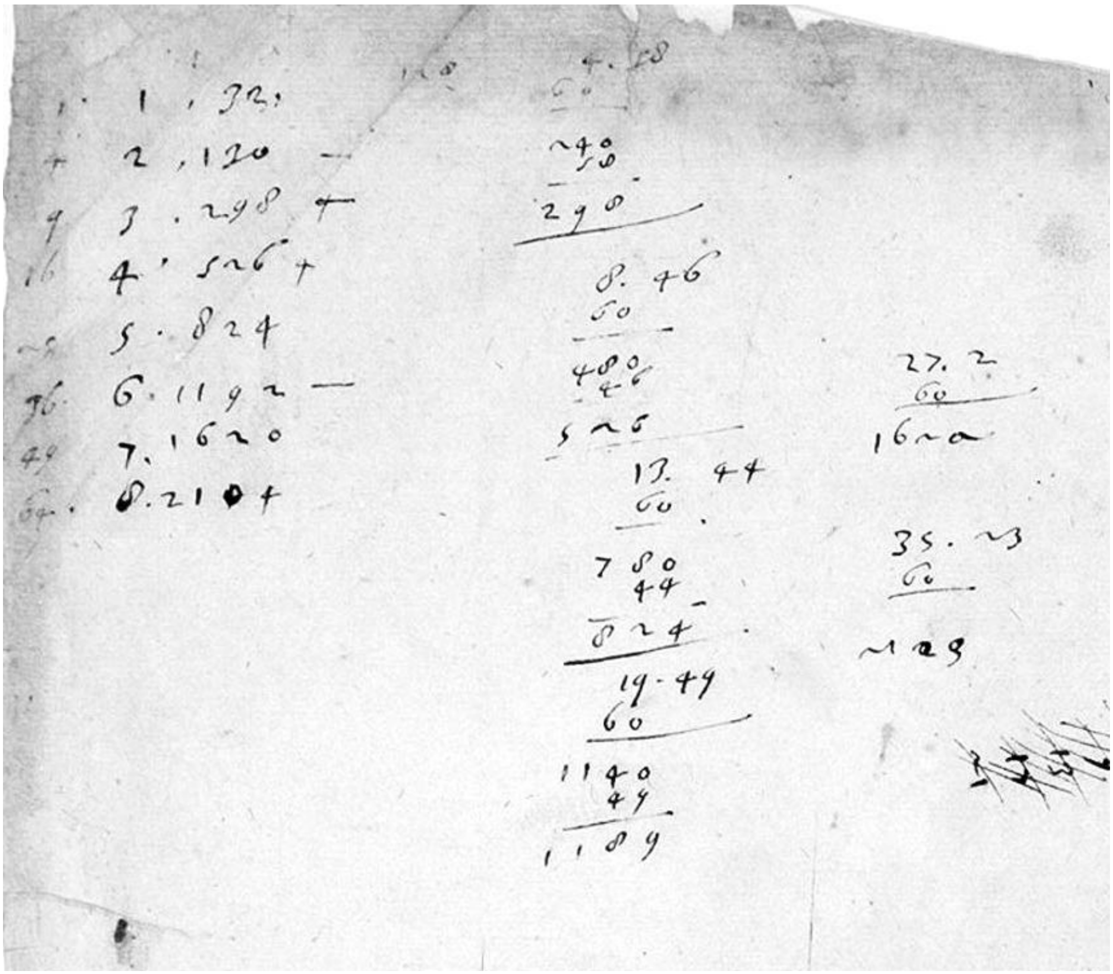

Fig. 3 sheet 107v

Sheet 107v, which was the subject of one of Drake's publications[9], is reproduced above. Why does Galileo used a 5.64-cm rule to measure lengths of up to 2 metres?

What accuracy might he hope to keep as he moved his mini-ruler as many as 36 times when measuring the greater length, equal to 2104 points,

[9] Drake, "The Role of Music in Galileo's Experiments," (cit. n.5). By the same author, see also *Galileo - Una biografia scientifica*, (Bologna: Il Mulino, 1988), pp.134-138. In a footnote on p. 136, Drake remarks that measurements taken by himself in Toronto and by Ben Rose in New York, using a metronome, were less accurate because "the concurrence of two external sounds is much more difficult to assess than the concurrence of one such sound with a loud rhythm from firmly singing a jolly march".

and why did he attach such importance to 23 remaining points? For Drake, sheet 107v describes the results of the measurements on the inclined plane (Drake assumed the angle of inclination was 1.7°), on which the positions for 8 equal time intervals had been marked:

> The objective of this modernized test was to measure as precisely as possible the distances travelled from rest by a ball rolling down an inclined plane at the ends of eight equal times (in this case at .55-second intervals). The grooved inclined plane used in the reconstruction was 6 1/2 feet long and was set at an angle of 1.7 degrees; it was fitted with a stop at the higher end, against which the two-inch steel ball could be held. The time intervals were established by singing "Onward, Christian Soldiers" at a tempo of about two notes per second. At one note the ball was released, and the position of the ball at subsequent notes were marked with chalk. […] A rubber band was then put around the plane at each chalk mark. The position of the rubber bands were adjusted so that the audible bump made by the ball in passing each band would always come exactly as a note of the march.

Of course, it could be argued that the ruler was longer, but then why divide it into 60-point units?

The arguments I present in this paper on the measurement of the *punctus*, taking it as equal to *picciolo* = 1/240 of a *braccio fiorentino a terra*[10], i.e. 2.29 mm, further complicate the already daring interpretation of sheet 107v, along the lines drawn by Drake, even by introducing the corrections of the groove[11] that reduce the acceleration on the inclined plane by half and by limiting the inclination to an angle of 1.7°.

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[10] One *braccio fiorentino a terra* equals 550.63 mm, one *braccio di panno* equals 583.02 mm.

[11] $a = [g \text{ sen } \varphi]\,[1 - (c/d)^2]\,[7/5 - (c/d)^2]^{-1}$ is the value of acceleration on an inclined plane when the groove is wide "c" and the ball diameter is "d" an the angle of inclination is "φ". If d/c = 1.22 (as it is in the experiments that

Let's make a few calculations. The effective acceleration is g_{eff} = 0,131 ms^{-2} (supposing the ratio of the diameter of the sphere to the width of the groove is 1.22). If we introduce 2.29 mm for the point, the full path of 2104 points equals 4.818 m. The time interval is 8.58 seconds and, at the end of the path, the ball rolls at a speed of 1.12 m/s.

We can evaluate the uncertainty of the measurements. In Drake's opinion, the limit of 1/64 of a second was accessible to such a good lute player as Galileo to perceive differences between the sound of the passing ball and the song beat. The uncertainty for 1/64 of a second is 17 mm (approximately 8 points); the uncertainty is instead 112 mm (approximately 50 points) if the ability to distinguish different sounds is at 1/10 of a second, especially if such sounds are not perfectly clear. The conditions with which I have direct experience are somewhat worse. In Drake's opinion, the path is instead 1.978 m long, the time is 5.50 s and the final speed is 0.7 m s^{-1}. We have an uncertainty of 11 mm (12 points) if the ear can resolve 1/64 of a second. If instead the limit is 1/10 of a second, the uncertainty increases to 70 mm (74 points).

This contradictory situation was for me difficult to avoid, until I learned of a newer interpretation of sheet 107v, which had been presented in a recent work[12], the product of a cooperation between the Berlin Max Planck Institute for the History of Science and two Florentine partners: the Museo e Istituto di Storia della Scienza and the Istituto Nazionale di Fisica Nucleare. In the new essay, the table and the calculations of sheet 107v are associated with Galileo's attempt to empirically compare the parabola to the curve generated by a chain hanging under the effect of gravity. On the other side

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led him to conclude that a body falls from 100 *braccia* in 5 seconds), then, from g (Padova) = 9.8065 m s^{-2}, we obtain a = g sen φ [0.32813 / 0.72814] = 9.8065 x 0.4506 sen φ = 4.419 sen φ = 4.419 x 0.0297 = 0.131 ms^{-2}.

[12] Jürgen Renn, Peter Damerow, Simone Rieger, and Michele Camerota, "Hunting the white elephant: when and how Galileo discover the laws of fall," (Berlin: Max Plank Institute for the History of Science, 1998)., preprint n° 97.

of the sheet, referred to as[13] 107r, the two curves have one end in common and reach the same lower level. One of the curves symmetrically continues back to the starting level, while the other curve stops at the lower point.

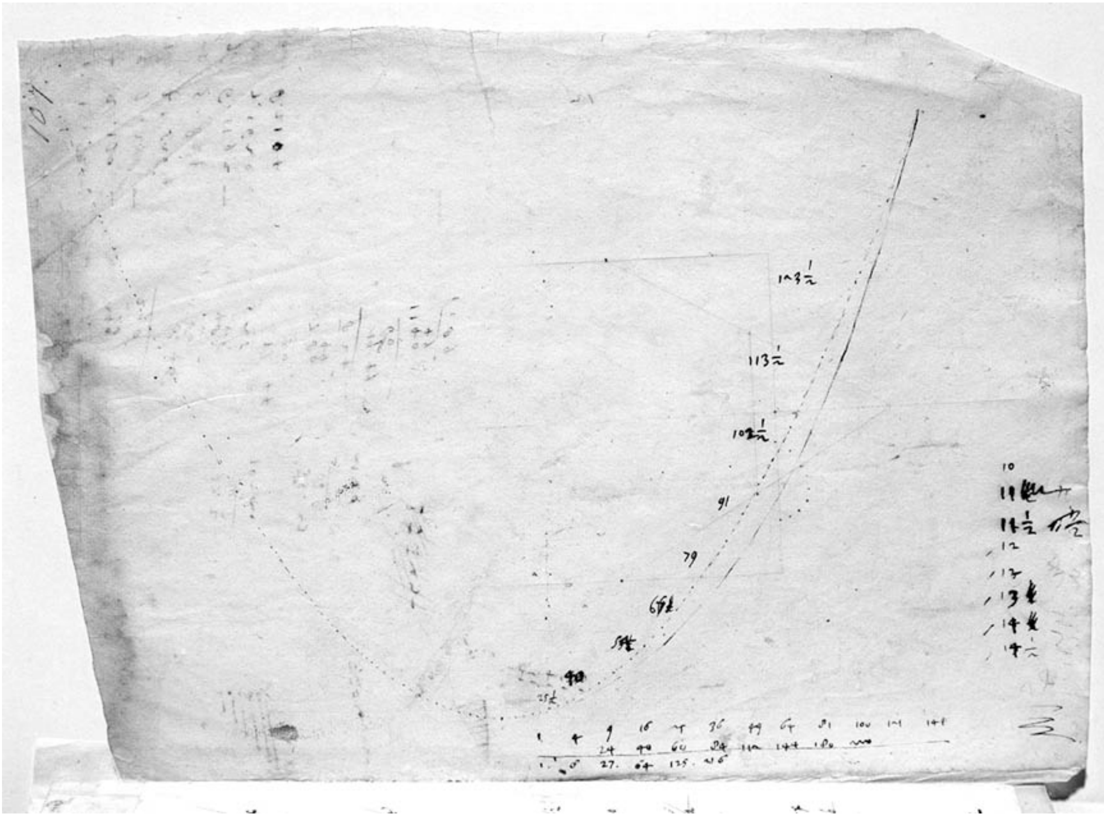

Fig. 4 sheet 107r

While I reproduce this sheet as well, I add that the authors claim their interpretation is “incompatible with Drake’s interpretation of sheet 107v”. However, the authors seem to maintain Drake’s geometrical quantities:

> The measure used by Galileo was probably the “punto”, so that the experiment must have been performed with a hanging chain of about 24 meters wide and 2 meters high.

[13] Of all the sheets of manuscript 72, numbered by sheets, as usual we state the two sides of each sheet with the letters: r = recto and v = verso.

If we recall the properties of the catenary, that there exists a definite relation between the length and the width of the chain hanging from its ends, the authors claim that if the maximum depth is as indicated by Galileo - 2123 - then the distance between the suspension points must be about 24000, and these measurements give an angle of approximately 20 degrees with respect to the horizontal at the suspension point. A 1:20 proportion in the two measurements of the catenary is compatible with one of the symmetrical curves shown in sheets 40 and 41.

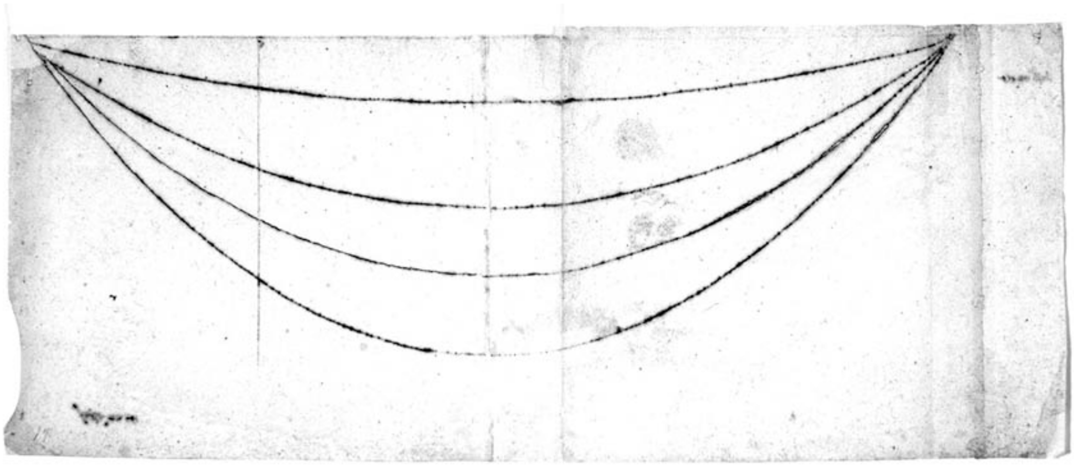

Fig. 5 sheet 42r

The one below is a catenary (a *catenella* for Galileo) with a steeper angle.

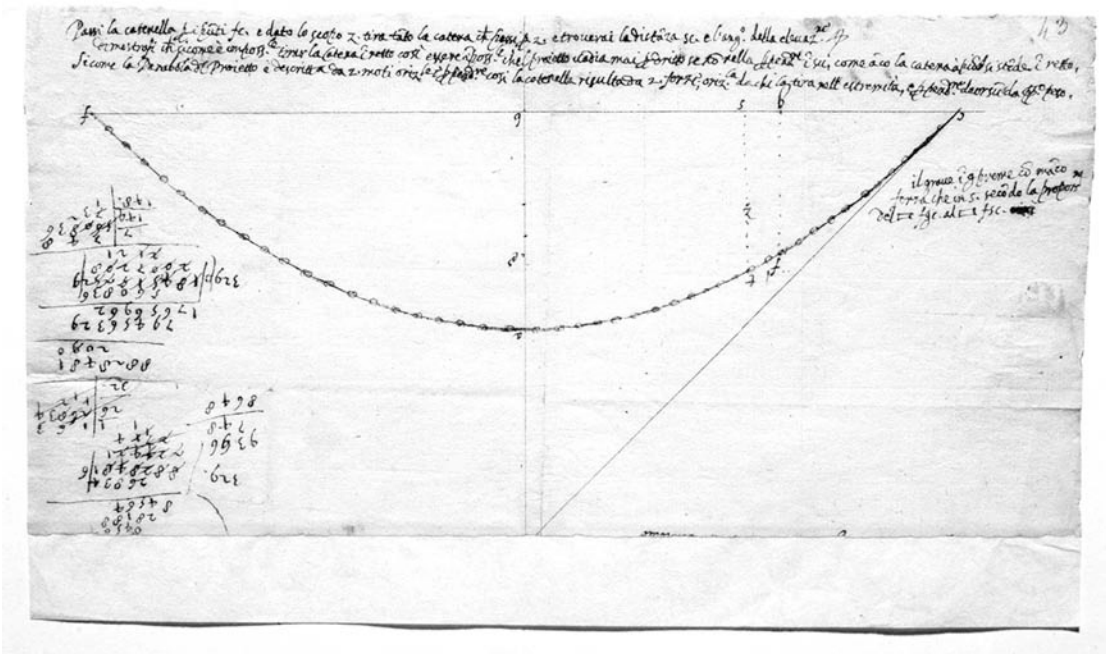

Fig. 6 sheet 43r

According to Renn, Damerow, Rieger and Camerota[14], the following scenario seems to be the most plausible: Galileo measured the vertical height of a hanging chain at eight points at regular distances from the lowest point of the curve. «Assuming that the highest value of 2123 in the calculation for Galileo's table on folio page 107v is the maximal vertical measure of the hanging chain, - they write - the best approximation is achieved for a width of about 24000». If these numbers are *puncta*, then are also *piccioli*. But 24.000 *piccioli* are 100 *braccia*, and a real chain of this lenght is hardly credible.

Virtually all scholars who have discussed the quantification of Galileo's *punctus* agree on a value of less than one millimetre, a completely arbitrary measurement with no relation with any fraction of the fundamental unit - the *braccio fiorentino* - regularly used by Galileo in his length measurements. For a broader view of the problem, we should investigate the historical context and the evolution of linear measurements in Italy, in particular in the Grand Duchy of Tuscany.

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[14] See footnote 11.

A recent publication[15] provides useful information on the *punctus* as one of the minor divisions of the linear measurement unit in the antique measurement systems of some Italian cities. We can separate the cities into those that fixed the *punto* above one millimetre,and those that fixed it at less than one millimetre. The table below lists the cities that gave a length of 2,6 mm to 4,1 mm for a *punto*. The number in brackets is the measurement of the *punto* in millimetres.

(2,639)	Bologna	(3,277)	Voghera
(2,753)	Cento	(3,302)	Brescia, Tortona
(2,805)	Ferrara	(3,307)	Novi Ligure
(3,022)	Milano e Pallanza	(3,358)	Cremona
(3,040)	Bergamo	(3,361)	Casale Monferrato
(3.099)	Sondrio	(3,435)	Massa
(3,133)	Como	(3,479)	Acqui
(3,162)	Lodi	(3,568)	Torino
(3,204)	Mortara	(3,632)	Modena
(3,261)	Piacenza	(3,644)	Cagliari
(3,262)	Crema	(3,786)	Parma
(3,270)	Novara	(4,101)	Lucca
(3,277)	Bobbio, Pavia	(5,846)	Ravenna

Six cities are an exception, with measurements far below one millimetre:

(0,143)	Genova e La Spezia	0,203)	Firenze
(0,144)	Porto Maurizio	(0,439)	Napoli
(0,149)	Palermo		

[15] ZUPKO RONALD EDWARD, Italian Weights and Measures from the Middle Ages to the Nineteenth Century, Philadelphia, 1981.

The author of this essay, Zupko, notes that some writers who spoke of the *punto* (G. Cardano, G. Cataneo, C.R. Guarini, H. Doursther, C. Salvati) stated that it is divided into 12 *atomi*, and so it is not always the smallest subdivision of the basic measurement unit used in these places. It is clear that with the advancement of technology it was useful to have smaller divisions of the basic measurement unit that were added with time. Note that in his *Practica Geometriae*[16], Leonardo Fibonacci had illustrated the subdivision of the linear measurements in the Pisan manner as follows:

> Ego vero, secundum pisanorum incedere volens consuetudinem, a pertica summam initium. Pertica pisana linealis, sex linealibus pedibus constat: pes vero linealis decem et octo punctis linealibus constat.

Thus in Pisa, the *punto*, the eighteenth part of a foot, was a fairly large unit, 15-18 millimetres. The *denarius* existed in the Pisan system as a surface measure, equal to one square foot and divided into *unciae*. Fibonacci actually continues with his presentation by introducing the *denarius* as follows:

> Item superficialis pertica continet in se denarios .xxxvi. de censura; et ita contingunt unicuique pedi denarij sex: et uncia superficialis est tertia pars denarij. Denarius quoque habet unum pedem in longitudine, et unum in latitudine; et ita denarius quadratus ex quatuor rectis constat angulis: et sic denarius est trigesima sexta pars totius perticae superficialis.

For Florence, we recall an event of importance for this story, when on July 11th 1782 Grand Duke Pietro Leopoldo issued a decree for sorting out the many different weights and measures that were then used in the cities,

16 *Scritti di Leonardo Pisano*…published by Baldassarre Boncompagni, Roma, 1857. Vol. II, *La Practica Geometriae di Leonardo Pisano secondo la lezione del codice urbinate n° 292 della Biblioteca Vaticana*, p. 3.

provinces, or communities of the Grand Duchy of Tuscany by printing the «TAVOLE DI RAGGUAGLIO per la riduzione dei pesi e misure che si usano in diversi luoghi del Granducato di Toscana al peso e misura veglianti in Firenze». The *picciolo* no longer appears in these measures, being replaced by the denarius, (*denarium* was the Roman silver coin, worth ten asses), whose etymology suggests a further division into ten *punti*, as was actually the case. The decree led to the removal from the official measures of the *braccio a terra*, of which the Tables said:

Il Braccio a Terra di Firenze diviso in Soldi 20, ed ogni Soldo in Denari dodici corrisponde a Braccia Fiorentine da Panno – Soldi 18, Denari 10 e 8/12.	The Florentine *Braccio a terra* divided into 20 *soldi,* and each *soldo* into twelve *denari,* equals in Florentine *Braccia da Panno – 18 Soldi, 10* and *8/12* Denari.

Thus, (*Braccio a Terra*) = 17/18 (*Braccio da Panno*). The unit *braccio a terra* is associated with geographical measurements, e.g. one mile equals 3000 such *braccia*. Galileo, in his *Dialogo sopra i due massimi sistemi del mondo*, starting from the repeated statement that a body falls from a height of one hundred *braccia* in five seconds, infers the time the same body would take to reach the earth from the moon by taking one Italian mile to be 3000 *braccia*.

Leonardo Ximenes, 25 years before the Grand Duke's reform, had compared the metric standards of Florence with the rectified *mezza tesa*[17] of the Royal Academy of Paris, which Monsieur de la Condamine brought when he visited Florence. The information he provides in a well-known paper[18] is extremely valuable. Here is how he describes the situation of "the

[17] The *toise* is a measure of six Parisian feet.

[18] XIMENES L., *Del vecchio e nuovo gnomone fiorentino e delle osservazioni, astronomiche fisiche ed architettoniche fatte nel verificarne la costruzione*. Firenze, 1757.

public standards, as we have in Florence, of the basic length measurements that are the *braccia*".

Io dico che quattro di questi ne abbiamo alle pubbliche Carceri, accanto alla porta che chiamasi *del fisco*, ed una al tribunale, che chiamasi *della parte*. Due campioni sono alla sinistra della porta del fisco, e due alla destra incastrati nella muraglia di pietra del Palazzo detto *del Bargello*. Il primo, e più basso è il Braccio, che volgarmente dicesi *Braccio a terra*, ed è solamente in uso nell'Agrimensoria; il secondo più alto è il Braccio, che domandasi *da panno*, e si adopera non solamente per la misura de' drappi, ma eziandio per tutti gli usi della Città; ed è il solo, che il volgo intenda, e conosca.	I say that four of these we have at the public *Carceri*, near the door called *del fisco,* and one at the *tribunale* called *della parte*. Two standards are on the left of the *porta del fisco,* and two on the right, set into the stonewall of *Palazzo del Bargello*. The former and lower one is the *braccio*, commonly known as *Braccio a terra*, and is only used in land surveying; the latter and higher one is the *braccio*, known as *Braccio da panno*, is used not only to measure cloth but also for all uses of the City; and it is the only one that the common people know and understand.

Ximenes then goes on to illustrate the problems he had faced withthe measurements using the Parisian standard and adds:

Ho osservato che nel braccio a terra vi erano alcune divisioni, che rappresentavano l'ottava parte, la quarta, la terza, e la metà. […] Onde, avendo presa la misura di questo braccio nel pian della striscia, l'ho ritrovato di *pollici 20, linee 4,*	I noticed the *braccio a terra* had some divisions that made up the eighth part, the fourth, the third and one half. […] Hence, as I had taking the measurement of this *braccio* on the strip plane, I found it measured *20 inches, 4 lines, 15 hundredths*, i.e. 244.15 Parisian lines. Mr

centesime 15, cioè di linee Parigine 244,15. Il Signor Giacomo Cassini parlando di questo stesso braccio, lo fa di linee 243,00. Onde vi si trova il divario assai considerabile di linee 1.15.

Il secondo braccio è il braccio da panno, il quale è diviso in più parti aliquote, che non è il primo. Poiché vi si scorge la parte sedicesima, l'ottava, la quarta, la terza, e la metà […] Il valore di questo braccio nelle parti del piè Parigino corrisponde a pollici *21. linee 6. centesime 40,* ovvero a linee 258,40.

[…] poiché in tutte le riduzioni, che gl'Ingegneri fanno dell'un braccio nell'altro, ed ancora in alcuni computi del Padre Abate Grandi, ed altri uomini di credito, si suppone la proporzione tra 'l primo, e secondo braccio come 17:18. Per la qual cosa supponendo il braccio a terra di linee 244.15, tornerebbe quello da panno di linee 258.51, cioè maggiore di 11 centesime di linea rispetto alla misura attuale; differenza assai tenue; e che nasce dalla

Giacomo Cassini, speaking of this *braccio*, took it to equal 243.00 lines. Hence the considerable difference of 1.15 lines.

The second *braccio* is the *braccio da panno*, which consists of several aliquot parts, differently from the first *braccio*. There are the sixteenth part, the eighth, the fourth, the third, one half. […] The value of this *braccio* in the parts of the Parisian foot corresponds to *21 inches, 6 lines, 40 hundredths,* i.e. 258.40 lines.

[…] since in all conversions the engineers change either *braccia* into the other and again in some calculations by Father Grandi and other men of authority, the ratio of the first *braccio* to the second *braccio* is taken as equal to 17:18. On these grounds, supposing the *braccio a terra* equals 244.15 lines, the *braccio da panno* would turn out to be 258.51 lines, that is, 11 hundredths of a line more than the current measure, a very small difference; and which comes from the difficulty of expressing either *braccia* in terms of the other. These two standards are equally old, equally authentic, equally well preserved. Hence, as there are no

difficoltà di limitare i termini nell'un braccio, e nell'altro. Ora questi due campioni sono ugualmente antichi, ugualmente autentici, ugualmente conservati. Onde, non essendovi maggior ragione per l'uno, che per l'altro, io piglierò la lor semidifferenza, la quale sottrarrò dal primo,ed aggiungerò al secondo; diminuendo di una millesima di linea, per mantenere più stretta la proporzione; e così sarà
il braccio a terra Fiorentino, rettificato di linee Parigine 244. 095 e il braccio da panno di 258. 454.

more grounds for preferring one over the other, I will take their semi-difference, which I will subtract from the first one and will add to the second one by reducing one thousandth of a line to keep the ratio closer: Thus the Florentine *braccio a terra*, corrected by Parisian lines, will be 244.095 and the *braccio da panno* 258.454.

As the Parisian line is 2.2558 mm long, we have the most certain measurements of the two Florentine standards that I chose:

Braccio da panno: 583.02 mm
Braccio a terra : 550.63 mm

Pietro Leopoldo's reform was followed by another promulgated on December 19th 1808 by the Council of Tuscany in the name of the Emperor of the French, King of Italy, *Protettore della Confederazione del Reno*, on the "weights and measures of the Empire and their use in the three departments of Tuscany effective as from January 1st 1810». The reform introduced new length measurements, including the metre, defined as the "fundamental unit of weights and measures: the ten millionth part of a quarter of the earth's meridian". Conversion tables were published, and they included the ratio of a *braccio* (*da panno*) to a metre,

1 *braccio* = 0.583626 m

a ratio that was fixed by the Weights and Measures Committee established by a decree of the Imperial Council on July 1st 1808. This ratio was never changed (see the table below).

Pietro Panini's notice, which introduces the recent collection of texts *del Cimento*[19], contains a conversion table (that has been reproduced online by the web page of the Istituto e Museo di Storia della Scienza of Florence[20])

[19] *Opere dei Discepoli di Galileo Galilei - L'Accademia del Cimento*, part one, Firenze 1942, p. 75.

[20] The address is:
http://brunelleschi.imss.fi.it/cimentosite/glossario_TAVOLE.html

Miglio	Braccia 2833 1/3		1653,607 m
braccio	20 soldi		58,3626 cm
soldo	12 denari	6 piccioli	2,9181 cm
quattrino	4 denari		0,9727 cm
denaro	12 punti		O,2432 cm
punto			0.0203 cm
Un braccio e 1/4			72,9532 cm
2/3 di braccio			38,9084 cm
16 soldi			46,69008 cm
3/10 di braccio	18 quattrini		17,50778 cm
3/4 di braccio	15 soldi		43,7718 cm
8 quattrini	1/15 di braccio		7,7816 cm

Strange enough, this table still mentions the *piccioli*.

This linear measurement unit was well known to the Accademici del Cimento. For instance, the aforesaid Florentine edition of the experiences[21] reads:

NUOVO MODO DI SPERIMENTARE LA MASSIMA COMPRESSIONE E DILATAZIONE DELL'ARIA

Facciasi prima una canna di vetro AC più soda e forte che si

NEW METHOD FOR TESTING THE AIR COMPRESSION AND EXPANSION LIMITS

First, make an AC glass pipe to be as strong and sound as possible, one and a half *braccio* long, and the bore of such pipe should be a *quattrino*

[21] *Opere dei Discepoli di Galileo Galilei - L'Accademia del Cimento*, part one, Firenze 1942, p. 421-422. It can also be found in GIOVANNI TARGIONI TOZZETTI, *Notizie degli Aggradimenti delle Scienze Fisiche accaduti in Toscana nel corso di anni LX del secolo XVII*, Volume II, part two, p. 507-509.

possa, lunga un braccio e mezzo, et il diametro della larghezza interna di detta canna di vetro sia un quattrino o pure 1/60 di braccio […]

[…] Per osservar poi la massima dilatazione dell'aria facciasi una canna di vetro AB lunga un braccio e mezzo, sturata in A, et il diametro della sua capacità sia di due piccioli, o pure 1/120 di braccio. Alla medesima canna di vetro sia continuata, mediante il collo di vetro BC, una palla o sfera di vetro CED, il cui diametro ED sia 18 quatrini o pure 3/10 di braccio, et alla parte inferiore di detta palla CED continui un cannello di vetro NO, e la sua estremità O sia diligentemente con lo stucco a fuoco unita saldamente al cannone di metallo FGH. Dovrà l'altezza FH di detto cannone 3/4 di braccio e la larghezza FG otto quattrini o 2/15 di braccio […]

or else 1/60 of an *braccio*. […]

[…] Then, to measure the air expansion limit, make glass pipe AB, one and a half *braccio* long, opening at A, and the diameter of its capacity should be two *piccioli* or else 1/120 of an *braccio*. The same glass pipe should continue, through the glass neck BC, into a glass ball or sphere CED, whose diameter ED should be 18 *quattrini* or else 3/10 of an *braccio*, and the lower section of said ball CED should continue into a glass shank NO, and its end O should be carefully welded with hot plaster to the metal tube FGH. The height of the tube should be 3/4 of an *braccio* and its width FG eight *quattrini* or 2/15 of an *braccio* […]

Here, a *picciolo* equals 1/240 of a braccio. Note the use of the word *quattrino*, meaning 1/60 of a *braccio*, i.e. 4 *piccioli*

After a while, this fraction of a *braccio* came to be called *denaro*, the name that replaced the *picciolo*, while the *punto* made its appearance as the 12th part of a *denaro*, or 1/2880 of a *braccio*.

This statement, in the notes of a scientific academy, proves that the *picciolo* existed in Florence in the XVII century and it was the smallest fraction of the *braccio*, its 240th part, and strongly corroborates the assumption that this measurement must have been translated by Galileo with the Latin word *punctus*, perfectly justified by the common usage in many Italian cities.

The letters of some scientists that I am going to use here will help unravel the situation of a measurement that is so unusual just because it is so small.

In a letter[22] from Giovanni Alfonso Borelli to Prince Leopoldo de' Medici, which bears no date but must have been written between 1656 (when he was called to the chair that had been held by his teacher, Castelli) and 1667 (when he went back to Messina), I find a *denaro* indicated as 1/60 of a *braccio*. Borelli had been asked to measure the amount of rainwater that could be collected in a vase having a mouth of one square *braccio*. He wrote to the Grand Duke's brother:

[…] E però, quando il semidiametro della bocca del catino sia 19 denari, o 19/60 di braccio, la metà della circonferenza del cerchio verrà ad essere 59 denari di braccio, e di più 69/100 di un denaro. Sicché la superficie, o area di tal cerchio verrà ad essere prossimamente1134 - 112/100 denari quadrati. E perché per tal superficie in due [*è stato letto 2*

[…] And yet, if the semi-diameter of the mouth of the bowl is 19 *denari* or 19/60 of an *braccio*, one half of the circumference of the circle will be 59 *denari* of an *braccio* and over 69/100 of a *denaro*. So, the surface or area of such circle will be approximately 1134 – 112/100 square *denari*. And, since two pounds of rainwater will flow through such surface over two [*has 2 been read instead of 5?*]

[22] KNOWLES MIDDLETON W.E., *Some unpublished correspondence of Giovanni Alfonso Borelli*, Annali dell'Istituto e Museo di Storia della Scienza di Firenze, Anno XI, 1984, pp. 98-132.

al posto di 5?] minuti d'ora passeranno due libbre d'acqua piovana, addunque supponendo uniformemente copiosa e frequente tal pioggia, in un'ora vi passeranno lib. 24 d'acqua. Dal che ne segue che in un vaso, la cui bocca sia un braccio quadro, cioè sia 3600 denari quadri, vi entreranno medesimamente in un'ora 77 lib. d'acqua piovana prossimamente.

minutes of an hour, supposing therefore the rain is copious and frequent, 24 pounds of water will flow through it in an hour. Hence, approximately 77 pounds of rainwater will flow into a vessel whose mouth measures one square *braccio*, i.e. 3,600 square *denari*.

In Borelli's time, *denaro* and *quattrino* were both names used to mean 1/60 of a *braccio*. Obviously, *quattrino* was more appropriate as a name, as it instantly suggested it was worth 4 *piccioli*.

But it's not just Borelli who used the *denaro* as 1/60 of a *braccio*. It also appears in a letter dated November 6th 1664 from Paolo Falconieri to Lorenzo Magalotti[23], to whom he related an experiment made with Giuseppe Campani using the latter's telescope:

Vengo dunque ora di osservare uno occhiale, il quale accomodato di notte alla distanza di braccia 100 [*m. 58,3*] era con tre diversi

I have, therefore, just returned from seeing a telescope which, adjusted for night use at a distance of 100 *braccia* [58,3 m]. was tested with three diverse oculare. With these it

[23] RIGHINI BONELLI M.L., VAN HELDEN A., *Divini and Campani: a forgotten chapter in the history of the Accademia del Cimento,* Annali dell'Istituto e Museo di Storia della Scienza di Firenze, Anno VI, 1981, pp. 67, .

acuti co’ quali si è provato uno oscuro di b.^{a} [*cm. 58*] più lungo di quello di S.A., e colla sua lente un terzo [*di braccio*] un soldo e due denari [*cm. 23*], si che come voi vedete accomodato di giorno a una distanza di 3. miglia [*km. 5*] poco vi potrebbe correre tra q.^{sto} et il maggiore de’ vostri […]

was a night-time telescope one *braccia* [58 cm] longer than that of His Highness, and with its [objective] lens only it was a third of a *braccio*, a *soldo*, and two *denari* [23 cm] [longer]. As you see, therefore, adjusted for day-time use at a distance of 3 miles [5 km], there can be little difference

Righini Bonelli and van Helden correctly evaluate the size of Campani’s lens as (1/3 + 1/20 + 1/30) of a *braccio* = 25/60 of a *braccio* = 23 cm, giving two *denari* the value of 1/30 of a *braccio*[24].

An extremely interesting text, also written by Magalotti, that compares the measurements of the Papal State with the Tuscan ones. It reads:

palmo di passetto romano è 1/3 di b.^{o} e un soldo in circa.

One *palmo di passetto* of Rome is about 1/3 of a *braccio* and one *soldo*

Here we understand that Magalotti uses the *braccio da panno*, because the *palmo romano* equals 22.34 cm and 23/60 of a *braccio da panno* equals 22.35 cm. But what is most interesting is an ensuing note:

[24] I find some inconsistency in this article, in a note on page 133, with what I have just stated because for the Tuscan measures the authors assume the usual division of the braccio into 20 soldi, the soldo into 12 denari and the denaro into 12 punti, with a punto being equal to 0.0203 cm.

Occh[iale] di palmi 16. once 1. ---- è b.º 6. soldi 3. p. 2 in circa	Telescope of 16 palm, 2 *once* is about 6 *braccia*, 3 *soldi* and 2 *parti*.

The Roman measure in metres is equal to 3.8933705 m, while b.º 6. *soldi* 3. p.2 is equal to 3.8892999 m + p.2; hence, p.2 = 0.00407 and therefore: p. = 2.035 mm, i.e. 1/240 of a *braccio*. I have no doubts that here p. is *picciolo*, but the authors, inexplicably, assume p. means *parti*.

I found a further indication that substantiates my assumption in a letter from Lorenzo Magalotti to Pierandrea Forzoni, dated December 20th 1664. I reproduce the passage of interest as published:

M'avvisa bene aver ritrovato, che gli occhiali de' quali si servì il Campani all'ultima osservazione fatta su' nuovi fogli, sono i medesimi per l'appunto dell'altra volta, sì che non dobbiamo tanto sbigottirci per la loro eccellenza, mentre quello che il buon D. Matteo ci ha apprestato per di u b.[raccio] e 19 soldi [m. 1.14]. debb'esser di b. 3 e 1/8 [m. 1,82]. Quello di b. 6 soldi 3 p.[unti] 2 [m. 3,59], di b. 6 1/2 [m. 3,79] e quello che ci faceva trasecolare di b. 8 e soldi 14 [m. 5,08] di b. 10 [m.5,84]	He informs me that he has discovered that the telescopes used by Campani for the last observations made on the new sheets are exactly the same as those used the other time. We do not need, therefore, to be so awed by their excellence, since that to which the good Don Matteo has given the measure of one *braccio*, 19 *soldi* [1,14 m] should be of 3 1/8 *braccia* [1,82 m]. That of 6 *braccia*, 3 *soldi*, 2 *punti* [3,59 m] should be of 6 1/2 *braccia* [3,79] and that of 8 *braccia*, 14 *soldi* [5,08 m], which caused amazement here, should be of 10 *braccia* [5,84 m]

Here Righini Bonelli and van Helden rightly used p. for *punti*, and also gave the *punto* its value of 1/240 of a *braccio*: actually, if we convert b.

6 soldi 3 p[unti]. 2 [m 3,59] it into 240ths, we have 1440 + 36 + 2 = 1478 *punti*. Let's make some more calculations with the *braccio da panno*: (0.58302 / 240) 1478 = 3.590 m. and we find out that they too made the same calculations. Obviously, the result in metres is the consequence of the authors having accepted a p.[unto] as being 1/240 of a *braccio*. What is notable in the letters is that "p." may indicate either *piccioli* or *punti*.

I think I have also found a graphic representation of the *punto* in Lorini's work[25], a drawing on page 8 with the measurement of the *piede veneziano* and the *mezzo braccio fiorentino* "with which measures all the drawings herein will be formed". The *mezzo braccio* contains 10 divisions that correspond to 10 *soldi*, and in addition at the beginning of the first division it provides two measures. The longer one is exactly one quarter of a *soldo*, while the smaller one, placed askew and preceded by a "p", is exactly one twelfth of the measure of a *soldo* and is therefore a *picciolo*.

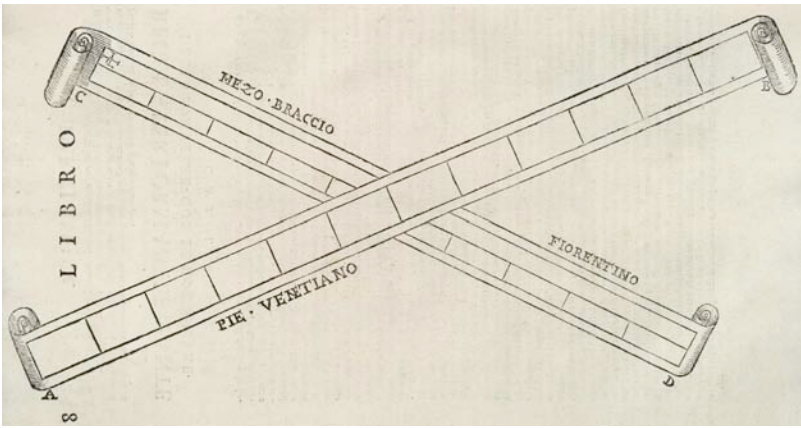


Fig. 7 Le fortificazioni di Buonaiuto Lorini, p. 8

If the foregoing is not sufficient evidence, I left for last the most convincing proof that before Pietro Leopoldo's reform the *picciolo* - not the *denaro* - was the basic sub-division of the *braccio fiorentino*. It is once again due to Leonardo Ximenes, who wrote in one of his books[26]:

[25] LE FORTIFICAZIONI DI BUONAIUTO LORINI, reprinted in Venice 1609, from Francesco Rampazetto.

[26] LEONARDO XIMENES, I sei primi elementi della geometria piana …, Venezia 1752 p.18-20.

Due braccia sono state in uso presso I vostri maggiori; le quali qualche autore malamente confonde. Il primo essi chiamarono *braccio da terra*, il quale essi nelle misure de' terreni, e nelle cose geografiche adoperavano. Il secondo chiamarono *braccio da panno*, perché appunto nel commercio, e nella stima de' panni era usato. Oramai a' giorni nostri questo secondo braccio è si comun divenuto, anche nelle misure terrestri, che il primo è quasi disusato. Pertanto questo braccio da Panno dividesi in 20 soldi, e ciascun soldo in 12 piccioli. Nella sua grandezza I geografi discordano moltissimo. Imperocché secondo lo Snellio sarebbe di particelle Parigine 2609, secondo il Riccioli di particelle 2550, secondo il Picard di 2580. Quest'ultima grandezza è la giusta, come ultimamente è stato ne' publici modelli ritrovato. Di questo braccio una quarta parte vi presenta la fig. 4. Per la notizia delle altre misure straniere io aggiungerò una tavoletta, nella quale la loro grandezza venga espressa in particelle Parigine, e in soldi, e piccioli del braccio Fiorentino.

Two different *braccia* have been used by your ancestors; which some authors badly mix up. The first one, they called *braccio da terra*, which they used to measure the land and geographical things. The second, they called *braccio da panno,* and it was used to estimate the measurements of cloths. Nowadays, this latter *braccio* has become so common, even in terrestrial measurements, that the former one is nearly out of use. Therefore, this *braccio da panno* is divided into 20 *soldi,* and each *soldo* into 12 *piccioli*. Geographers are in great disagreement over this measure. Since, according to Snellio it is 2609 Parisian parts, according to Riccioli 2550 parts, according to Picard 2580. The latter measure is the right one, as it has lately been found out from the public standards. Fig. 4 shows the fourth part of this *braccio*. For information on the other foreign measures, I will add a table in which their measures are given in Parisian parts and in *soldi* and *piccioli* of a Florentine *braccio*.

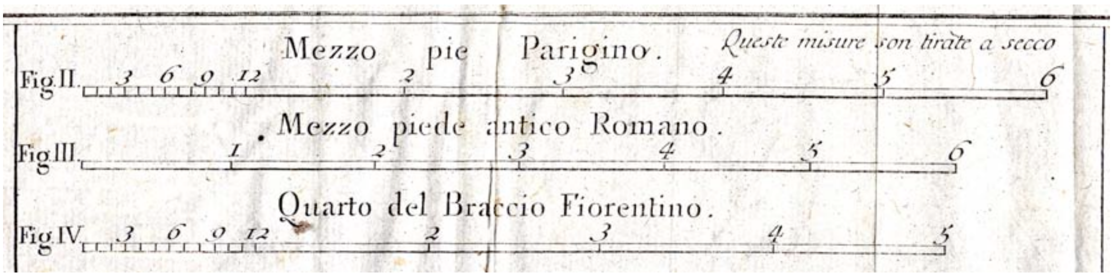


Fig. 8

RAPPORTI DI MISURE ESTERE
ALLE PARIGINE, E FIORENTINE

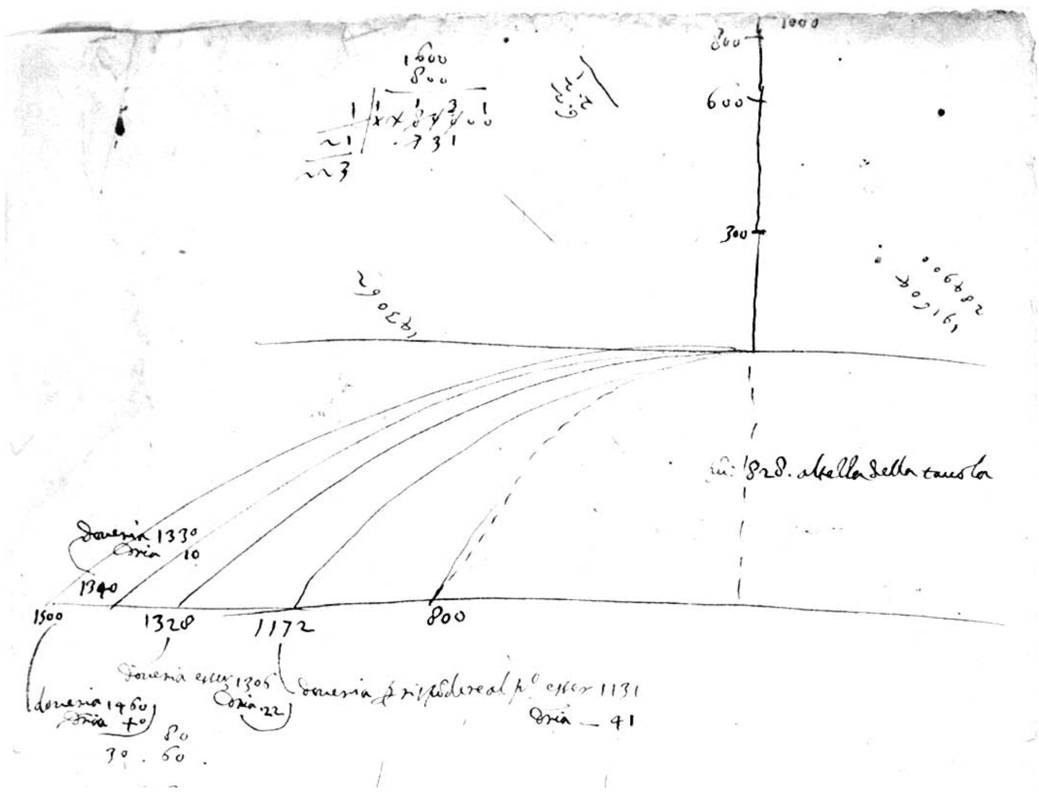
Differenti miſure Eſtere		Parti del Piè Parigino	Soldi	Piccioli	Frazioni di P.
Piede	Romano	1306.	10.	1.	21/43
	Greco	1360.	10.	6.	22/43
	Macedonico	1567.	12.	1.	33/43
	Di Londra	1351. 2/3	10.	5.	29/43
	Del Reno	1392.	10.	9.	28/43
	Spagnuolo	1240.	9.	7.	15/43
	Parigino	1440.	11.	1.	41/43
	Di Bologna	1682.	13.	---	20/43
	Di Leida	1390.	10.	9.	13/43
	Di Svevia	1316.	10.	2.	18/43
	Di Danzica	1272.	9.	10.	14/43
	D'Amſterdam	1252.	9.	9.	1/43
	D' Aliprando	1870. 1/2	14.	6.	--
Palmo	Di Napoli	1169.	9.	---	32/43
	Genoveſe	1113.	8.	7.	23/43
	Di Palermo	1073.	8.	3.	35/43
	Romano	990.	7.	8.	4/43
Piede	Naturale	1088.	8.	5.	9/43

fig. 9

A similar table, also containing the *braccio fiorentino da terra*, is shown in a booklet[27] which was published several times by Ximenes until 1751. That in Florence until the mid-eighteenth century a *picciolo* was 1/12 of a *soldo*, while a *denaro* or *quattrino* was 1/3 of a *soldo*, now seems substantiated. We note that the above fourth part of the *braccio fiorentino* is divided into 5 *soldi*. Only the first division is subdivided into 12 *piccioli*. The whole measure contains 60 *piccioli*, just the number found in the calculations of the sheet 107v.

But is the *picciolo* Galileo's *punctus*? The next Section will try to answer this question.

2. *The punto in Galileo's autograph papers*



[27] *Notizia de' Tempi ad uso degli Eruditi italiani e de' Viaggiatori per l'anno 1752. Al meridiano fiorentino. Alla quale si aggiungono alcune Tavole Geografiche, ed astronomiche, alcune osservazioni Astronomiche, e Geografiche dell'Anno scorso 1751, ed una breve dichiarazione degli usi di questa Operetta*, In Firenze 1571.

Fig. 10 sheet 116v

In order to reconstruct the experiments described on sheet 116v of Mss 72, which has been reproduced above, we need to know the measurements of the *punctus* that have been used to record the measurements of the initial specifications and the results of the throws.

Can evidence of the measurements of the *punctus* be found in Galileo's papers? Unfortunately, no accurate references have been found in Galileo's printed works and manuscripts. Two sheets of mss. 72, marked as 115v and 189v, have attracted the attention of some scholars.

Before getting to the heart of the subject, I will write out some passages from the *Dialogo* that will be useful later on. On *Second Day*, Galileo, wanting to counter Locher and indirectly Scheiner[28], - who had reckoned a ball falling from the moon onto the earth would take "much more than 6 days" - calculated this interval using his law: «the acceleration of the rectilinear motion of the bodies follows odd numbers *ab unitate*», and found a result of just three hours, 22 minutes and 4 seconds.

[…] la distanza dal concavo lunare al centro della Terra, che è miglia 196 000, facendo la distanza del concavo 56 semidiametri terrestri, come fa l'autore moderno, ed il semidiametro della Terra 3500 miglia di braccia 3000 l'uno, quali sono le nostre miglia italiane, […] facendo il computo sopra l'esperienza e	[…] the distance from the hollow of the moon to the centre of the Earth, which is 196,000 miles, taking the distance of the hollow as 56 semi-diameters of the Earth, as modern authors do, and the semi-diameter of the Earth as 3500 miles of 3000 *braccia* each, as our Italian miles are, […] if we calculate from experience and not with our fingers, we would cover the distance in

[28] Christoph Scheiner was the real author of that book, *the Disquisitiones Mathematicae de controversiis et novitatibus astronomicis* …, (Ingolstadt 1614) signed by his pupil, Johannes Gregorius Locher.

non su per le dita, si passerebbe in assai meno di 4 ore; e facendo il computo esatto, si passa in ore 3, minuti primi 22 e 4 secondi.

[…] Però avendo (come ho detto) con diligente esperienza osservato come un tal mobile passa, cadendo, l'altezza di 100 braccia in 5 secondi d'ora, diremo: se 100 braccia si passano in 5 secondi, braccia 588 000 000 (che tante sono 56 semidiametri della terra) in quanti secondi si passeranno? La regola per quest'operazione è che si multiplichi il terzo numero per il quadrato del secondo; ne viene 14 700 000 000, il quale si deve dividere per il primo, cioè per 100, e la radice quadrata del quoziente, che è 12 124, è il numero cercato, cioè 12 124 minuti secondi d'ora, che sono ore 3, minuti primi 22 e 4 secondi.

[…*l'operazione*] è assai facile. Segniamo questi tre numeri con le lettere A primo, B secondo, C terzo; A, C sono i numeri de gli spazii, B 'l numero del tempo: si cerca il

much less than 4 hours; and if we make an accurate calculation, we would cover the distance in 3 hours, 22 minutes and 4 seconds.

[…] Yet, as I have (as I said) very carefully observed how such a body moves as it falls, through a height of 100 *braccia* in 5 seconds, we can say: if 100 *braccia* can be covered in 5 seconds, how many seconds would it take to cover 588,000,000 *braccia* (which are equal to 56 semi-diameters of the Earth)? The rule for this operation is to multiply the third number by the square of the second one; the result is 14,700,000,000, which is to be divided by the first one, i.e. by 100, and the square root of the quotient, which is 12 124, is the number we are looking for, i.e. 12 124 seconds of an hour, which are 3 hours, 22 minutes and 4 seconds.

[…*this operation*] is very easy. Let's mark these three numbers with the letters A for the first one, B for the second one, C for the third one; A and C stand for space, B for time; we need to find the fourth number which also stands for time. And, since we know that the square of time B must be to the square of the time we are looking for as space quarto numero, pur del tempo. E perché noi sappiamo, che qual proporzione ha lo spazio A allo spazio C, tale deve avere il quadrato del tempo B al quadrato del tempo che si cerca, però, per la regola aurea, si multiplicherà il numero C per il quadrato del numero B, ed il prodotto si dividerà per il numero A, ed il quoziente sarà il quadrato del numero che si cerca, e la sua radice quadrata sarà l'istesso numero cercato.

A is to space C, though under the golden rule we must multiply the number C by the square of the number B and divide the product by the number A, and the quotient will be the square of the number we are looking for, and its square root will be the number we are looking for.

If one knows the time it takes to fall in a given space (for instance, 5 seconds in 100 *braccia*), one can find the time of any other falling distance through the golden rule that Galileo has so clearly explained and frequently used. Sheet 115v reads:

tempus totius diametri est 280 cum eius longitudo fuerit p. 48143
g^{o}. [= ergo] tempus diametri cuius long.do 4000 erit 80 2/3

The numbers provided in this sheet can also be found in the sheet 189v and therefore form a significant connection between the two papers although the abbreviation p. before the number 48143 appears only here. The lengths are therefore measured in *puncta*.
Let's consider the following relations between the lengths and the times in this fragment:

$(280)^2 : 48143 \quad = 1{,}6284\ldots$

$(80\ 2/3)^2 : 4000 = 1{,}6268\ldots$

The result suggests the presence of a constant of the motion. When a length is related by Galileo to a squared time, there are two possibilities: a falling motion or the oscillations of a pendulum. In principle, we cannot say to which case these time/length relations refer, but based on the magnitudes I think the latter case can be ruled out. If this is a natural falling motion, the value of the acceleration constant g/2 = 1.6284 is given, unfortunately, in unknown length (48143) and time (280) units.

If a *punto* equals 0.94 mm, the currently accepted value, then 4000 *punti* = 3.760 m and 48143 *punti* = 45.254 m

If, instead, a *punto* equals 1/240 of *a braccio a terra*[29], i.e. 2.29 mm, then the two lengths equal 4000 *punti* = 9.160 m and 48143 *punti* = 110.247 m

Why are the lengths called *diameter*? In sheet 189v, the length 4000 is just the measurement of the diameter of a circle. The properties of the inclined plane are often demonstrated by inscribing it as a chord of a circle whose vertical diameter is a length run through by free-falling bodies, and other straight paths are studied in configurations showing a circle and its diameter, whose falling time is sometimes given by comparison with the descent of the other paths.

A circumference with a vertical diameter having *a b* as its extremes and run through by the number 4000 is drawn in the top upper corner of sheet 189v. It also shows a chord having *bd* as its extremes. Further down it reads:

[29] Note that the *braccio a terra* di Firenze is worth 0.55063 metres, while the *braccio da panno* is worth 0.58302 metres.

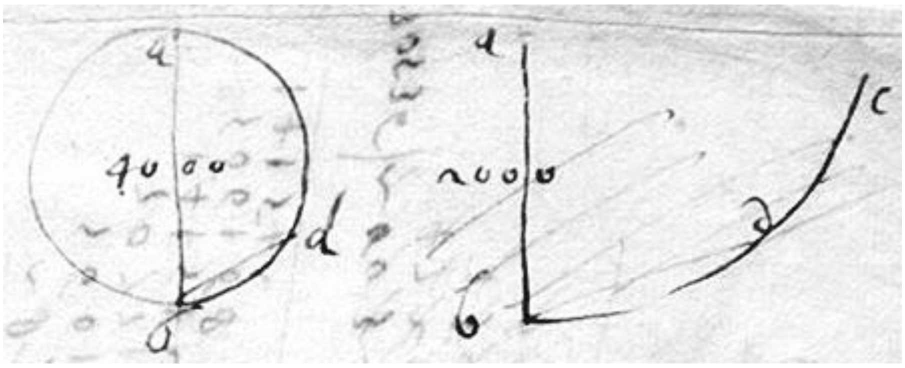


Fig. 11 sheet 189v

Si diameter *ab* sit 4000 arcus *bd* conficit temporibus 62

Perpendicularis cuius longitudini 48143 conficit temporibus 280

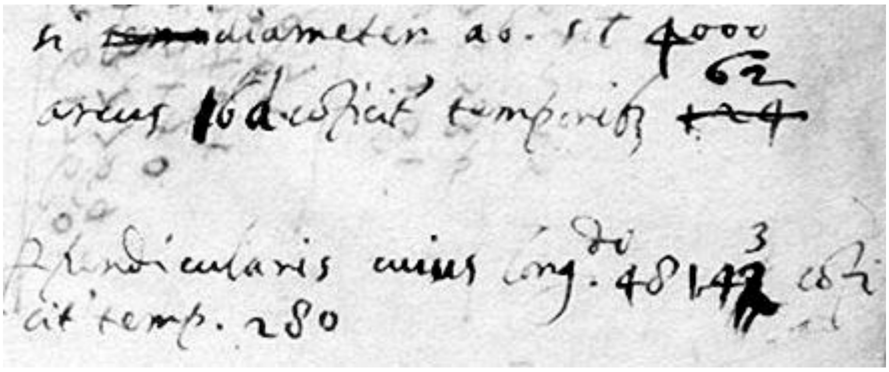


Fig. 12 sheet 189v

Note that Galileo wrote arc. Had he written *chord*, according to the theorem of the conjugate chords, he should have given the chord *bd* a time 80 2/3, which is the free-fall time along the diameter of 4000 units. Since he wrote *arcus*, we can only think of the pendulum. He drew on the top right corner a semi-circumference that then he deleted, correcting the sentence below; perhaps he meant to separately address the fall from 4000 *punti* and the motion of a pendulum, 2000 *punti* in length. We will come back to the assumption of the pendulum shortly.

In the middle of the page the text reads[30]:

[30] I do not understand where the numbers 670000000 and 27834 come from, but it is worth noting that (2 x 670000000):27834 = 48142.5; I do not think

bd	*ab*
27843	100000

Farther down, the number 100 000 x 6700 = 670 000 000 is divided by 27834 with a result of 24071.

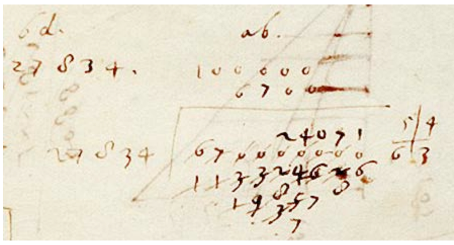


Fig. 12 sheet 189v

Still on sheet 189v, three heights are marked along a vertical straight line, at the lower left corner.

4000 13823 media 48143 in temp. 280

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it is a coincidence if it very close to 48143 which – please note – at first had been written as 48142. Obviously, we also have 24071 x 2 = 48142.

Another coincidence is given by 27834:2 =13917 which is very close to 13877.

The following relations may be significant: punti (4000:240) = *braccia* 16 2/3; punti (48143:240) ≈ *braccia* 200, $[280 :(80\ 2/3)]^2 \approx 12$; 48143:4000 ≈ 12.

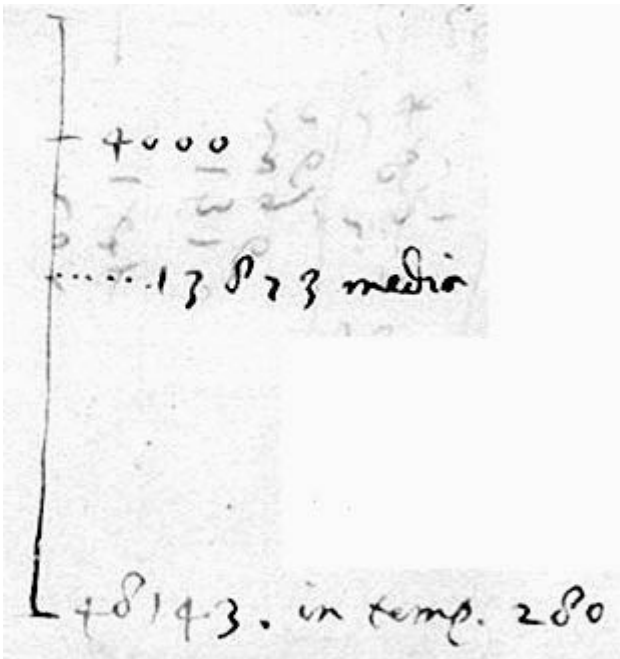

Fig. 13 sheet 189v

Across it, there is the operation used by Galileo to calculate the mean value.

Averaging means finding the unknown in the relation 48143:x = x:4000, and the result is x = 13877. In making this calculation, Galileo made a mistake, since he wrote 48143 x 4000 = 192472000 and therefore he obtained $(192472000)^{1/2}$ = 13873 as the mean.

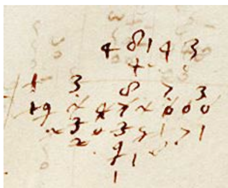

Fig. 14 sheet 189v

Taken together these operations and the sentence on sheet 115v, are the application of the golden rule that is so thoroughly discussed in the passage from the *Dialogo* reproduced at the beginning of this paragraph.

The rule is also presented in different words in one of the first corollaries of *Third Day* of the *Discorsi*[31].

> Secondly, we find that if we take, starting from the beginning of motion, any two spaces covered in any time, the relevant times are to each other as either space is to the proportional mean between the two given spaces.

If we follow the instructions provided in the corollary, we find

280 : 80 2/3 = 3.471
48143 : 13873 = 3.470

Supposing the numbers 4.000 – 13823 – 48143 are the heights reached when falling from a stationary position, then the times they take[32] are 80 2/3 – 150 - 280.

These figures show that the values of the *tempus* is not seconds or some count of the oscillations of a pendulum; rather, they seem to be numbers calculated by weighing the water collected by the water clock. In this case, the weight is likely to have been given in 1/60 of a *grano* (1 *grano* = 49.12 mg).

Let's go back to the assumption of the pendulum. To make the calculations in old units, I will express the acceleration in Padova in *braccia* and in *punti*, leaving open the option of the *braccio fiorentino*. For the *punti*, I will assume: *braccio* = 240 *punti*.

g = 9.8065 m/s = 17.8096 *braccia* s^{-2} = 4274 *punti* s^{-2} (for a *braccio a terra*)

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[31] G.G., vol. VIII, *Discorsi intorno a due Nuove Scienze*, p. 214: «Colligitur, secundo, quod si a principio lationis sumantur duo spatia quaelibet, quibuslibet temporibus peracta, tempora ipsorum erunt inter se ut alterum eorum ad spatium medium proportionale inter ipsa».

[32] Time up to 13823 = $[(80\ 2/3)^2\ 13823/4000]^{1/2}$ = 149.9…

$g = 9.8065$ m/s = 16.8201 *braccia* s^{-2} = 4037 *punti* s^{-2} (for a *braccio da panno*)

A 2000-*punti*-long pendulum has a period of approximately 4.3 seconds.

$T = 2\pi\ (2000/4274)^{1/2} = 4.298$ s (for a *braccio a terra*)
$T = 2\pi\ (2000/4037)^{1/2} = 4.422$ s (for a *braccio da panno*)

The arc *bd* covered by the pendulum corresponds to a quarter of a full oscillation, and the associated time is approximately 1 second.

$(4.298:4) = 1.0745$ s (for a *braccio a terra*)
$(4.422:4) = 1.1055$ s (for a *braccio da panno*)

Galileo did not know the value of gravitational acceleration but had directly weighed the water flowing out after a given number of oscillations of the pendulum. He had found that his way of measuring time by weighing water was very accurate when the time interval to be measured was a few seconds, because the amount of water flowing out of a *sottil cannellino (a thin tap)* did not alter the level of the water contained in the *gran vaso (big vessel)* and therefore the weight of the collected water was proportional to the time that had passed. For his researches, he did not need to know the period of the pendulum, even if he was well aware that he could convert the height of the water into time if necessary with the method he had illustrated on so many occasions.

The assumption of measurements having been taken with the water clock is substantiated if the water collected in 16 simple vibrations of the pendulum is the series of numbers that are written on the top left section of the sheet[33].

The addition 13 + 530 + 320 + 180 + 95 + 320 + 530 = 1988, divided by 16 simple oscillations, fixes the

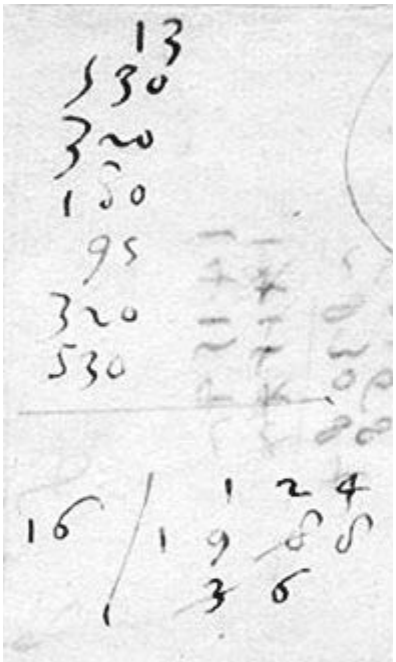

Fig. 14 sheet 189v

weight of the water flow out in one simple oscillation, which is 124.25 grani of water, 1/2 simple oscillation = 62.125 grani (almost the same words as those used by Galileo: «arcus *bd* conficit temporibus 62»). Time is measured by collecting water, and the final weight is 62.125 grani. We can calculate at once the weight of the water that has flown out in one second[34].

1 second =

62.125:1.074 = 57.84 *grani* of water = 2,84 g (for a *braccio a terra*)

62.125:1.106 = 56.17 *grani* of water = 2,76 g (for a *braccio da panno*)

~~35 g~~

We have thus obtained the correspondence between the time and the weight of the water.

To solve the problem of the *punto* once and for all, what we would need is having, for instance, the weight of one cube of water with the side measured in *punti*. By comparing it with the weight of one cube of water

[33] I think S. Drake has been the first to suggest this assumption.

[34] The flow of water through a small hole of 2 mm diameter is 4,35 gs^{-1} when the water level is 17 cm.

with one-millimetre sides, finding the relation between a *punto* and a millimetre would be a matter of seconds.

The consequences of these results is interesting and disappointing at the same time:

a) I am now certain that Galileo used a water clock to measure the time taken by free-falling bodies through experiments that were independent of those made on the inclined plane;

b) I am unable to decide on the size of the *punto* based on this type of indirect information.

Why, if Galileo knew the relation between space and time in free falling, did he write 100 *braccia* in 5 seconds instead of 3.44 seconds so many times in his *Dialogo*? Galileo already explained this to Giovanbattista Baliani: the one written in the *Dialogo* was not a direct measurement but it had been calculated from the measurements taken with the water clock by making a ball roll along a groove of the inclined plane. Galileo did not know how to correct the measurements from the effect of rotation. Now, we know that, on the inclined plane, where the ball diameter is d, the groove width is c, the plane length is L and the plane height is H, the effective acceleration[35] can be calculated from the motion formula. If $s(t) = (1/2)\ g\ t^2(h/L)\ [1 - (c/d)^2][7/5 - (c/d)^2]^{-1}$ is the space covered in the time t, then $g_{eff} = g\ [1 - (c/d)^2][7/5 - (c/d)^2]^{-1}$.

If $d/c = 1.218$ then $g_{eff} = 0.44903g = 4.40504\ ms^{-2} = 8$ braccia s^{-2} = 1920 *punti* s^{-2}, $24000 = (1/2)gt^2$ and $t = (48000/1920)^{1/2} = 5$ s

With d/c = 1.218 we obtain 100 *braccia* in 5 seconds. This is the relation between d and c and is precisely the result of this statement by Galileo.

The answer to the question we had asked ourselves is this: while he was writing the *Dialogo*, Galileo thought that the experiments on the plane were more accurate, more easily reproducible, more varied. Later, based on

[35] In the experiments I made in 1992, I found that acceleration is further reduced by the special friction of the groove, which reduces the calculated acceleration by approximately 15%. This affects the d/c relation.

direct measurements, he wanted to correct himself and he wrote in his own hand on the sides of the specimen of the *Dialogo sopra i due Massimi Sistemi del Mondo*, which is kept at the Seminar in Padova: «it is seen, in four pulses, to have passed over 100 *braccia* of space[36]».

Dipartimento di Fisica "Enrico Fermi"
Università di Pisa (Italy)
Istituto Nazionale di Fisica Nucleare (INFN)
Sezione di Pisa

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[36] G.G,, vol. VII, p. SIMP. «Di grazia, sia conceduto alla mia poca pratica nelle scienze matematiche dir liberamente come i vostri discorsi, fondati sopra proporzioni maggiori o minori e sopra altri termini da me non intesi quanto bisognerebbe, non mi hanno rimosso il dubbio, o, per meglio dire, l'incredulità, dell'esser necessario che quella gravissima palla di piombo di 100 libre di peso, lasciata cadere da alto, partendosi dalla quiete passi per ogni altissimo grado di tardità, mentre si vede in quattro battute di polso aver passato piú di 100 braccia di spazio: effetto che mi rende totalmente incredibile, quella in alcuno momento essersi trovata in stato tale di tardità, che continuandosi di muover con quella, non avesse né anco in mille anni passato lo spazio di mezo dito. E pure se questo è, vorrei esserne fatto capace».
(«Pray, let my poor experience in mathematic science freely say how your arguments, based on bigger or smaller ratios and about other terms that I have not understood as I should have, have not dispelled my doubts or rather my disbelief, that it is necessary that that very heavy ball of lead, 100 pounds in weight, if dropped down, from a state of rest passes through every state of slowness, while you see it has covered over 100 *braccia* of space in four pulse beats: an effect that makes it hard to believe that it may have ever been in such any state of slowness that while keeping moving with that it had not covered the space of half a finger even in one thousand years. And yet, if it is so, I would like to get to understand it».